AQA Maths Pure Core 2

Past Paper Pack

2006-2013

General Certificate of Education
January 2006
Advanced Subsidiary Examination

ASSESSMENT and QUALIFICATIONS ALLIANCE

MATHEMATICS **MPC2**
Unit Pure Core 2

Tuesday 10 January 2006 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 Given that $y = 16x + x^{-1}$, find the two values of x for which $\dfrac{dy}{dx} = 0$. *(5 marks)*

2 (a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for

$$\int_0^4 \frac{1}{x^2 + 1}\,dx$$

giving your answer to four significant figures. *(4 marks)*

(b) State how you could obtain a better approximation to the value of the integral using the trapezium rule. *(1 mark)*

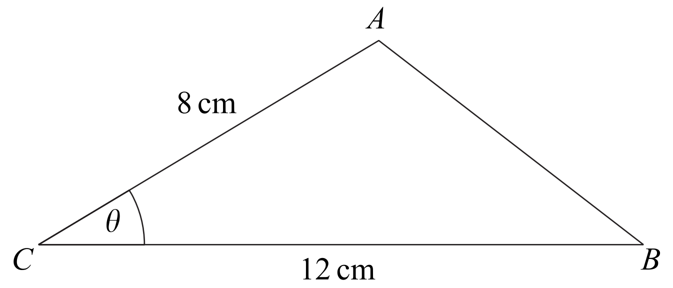
3 (a) Use logarithms to solve the equation $0.8^x = 0.05$, giving your answer to three decimal places. *(3 marks)*

(b) An infinite geometric series has common ratio r. The sum to infinity of the series is five times the first term of the series.

(i) Show that $r = 0.8$. *(3 marks)*

(ii) Given that the first term of the series is 20, find the least value of n such that the nth term of the series is less than 1. *(3 marks)*

4 The triangle ABC, shown in the diagram, is such that $AC = 8$ cm, $CB = 12$ cm and angle $ACB = \theta$ radians.

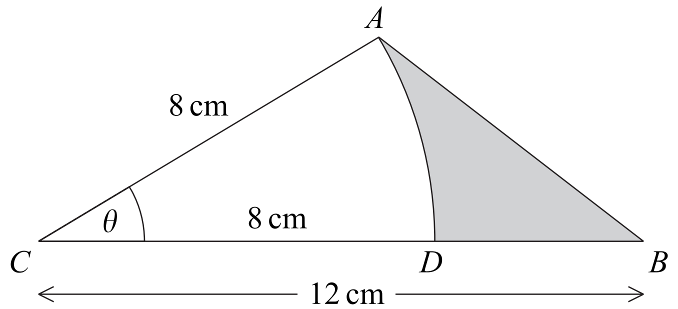


The area of triangle $ABC = 20\text{ cm}^2$.

(a) Show that $\theta = 0.430$ correct to three significant figures. *(3 marks)*

(b) Use the cosine rule to calculate the length of AB, giving your answer to two significant figures. *(3 marks)*

(c) The point D lies on CB such that AD is an arc of a circle centre C and radius 8 cm. The region bounded by the arc AD and the straight lines DB and AB is shaded in the diagram.



Calculate, to two significant figures:

(i) the length of the arc AD; *(2 marks)*

(ii) the area of the shaded region. *(3 marks)*

5 The nth term of a sequence is u_n.

The sequence is defined by

$$u_{n+1} = pu_n + q$$

where p and q are constants.

The first three terms of the sequence are given by

$$u_1 = 200 \qquad u_2 = 150 \qquad u_3 = 120$$

(a) Show that $p = 0.6$ and find the value of q. *(5 marks)*

(b) Find the value of u_4. *(1 mark)*

(c) The limit of u_n as n tends to infinity is L. Write down an equation for L and hence find the value of L. *(3 marks)*

6 (a) Describe the geometrical transformation that maps the curve with equation $y = \sin x$ onto the curve with equation:

(i) $y = 2\sin x$; *(2 marks)*

(ii) $y = -\sin x$; *(2 marks)*

(iii) $y = \sin(x - 30°)$. *(2 marks)*

(b) Solve the equation $\sin(\theta - 30°) = 0.7$, giving your answers to the nearest 0.1° in the interval $0° \leqslant \theta \leqslant 360°$. *(3 marks)*

(c) Prove that $(\cos x + \sin x)^2 + (\cos x - \sin x)^2 = 2$. *(4 marks)*

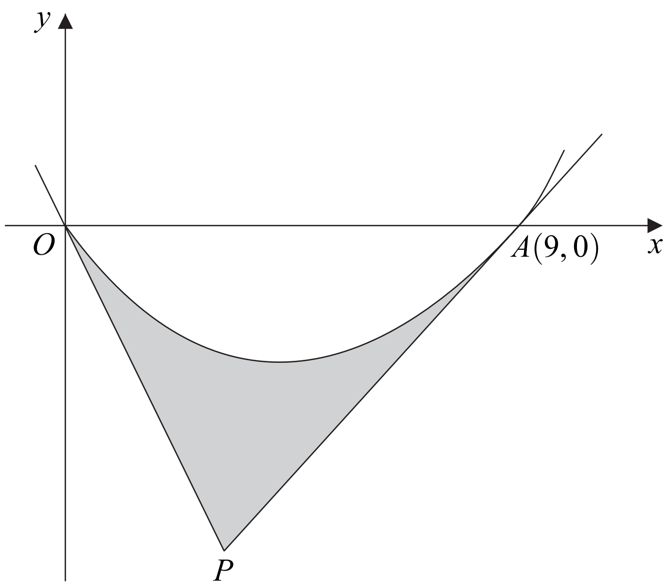
7 It is given that n satisfies the equation

$$2\log_a n - \log_a(5n - 24) = \log_a 4$$

(a) Show that $n^2 - 20n + 96 = 0$. *(3 marks)*

(b) Hence find the possible values of n. *(2 marks)*

8 A curve, drawn from the origin O, crosses the x-axis at the point $A(9, 0)$. Tangents to the curve at O and A meet at the point P, as shown in the diagram.



The curve, defined for $x \geqslant 0$, has equation

$$y = x^{\frac{3}{2}} - 3x$$

(a) Find $\dfrac{dy}{dx}$. *(2 marks)*

(b) (i) Find the value of $\dfrac{dy}{dx}$ at the point O and hence write down an equation of the tangent at O. *(2 marks)*

(ii) Show that the equation of the tangent at $A(9, 0)$ is $2y = 3x - 27$. *(3 marks)*

(iii) Hence find the coordinates of the point P where the two tangents meet. *(3 marks)*

(c) Find $\int \left(x^{\frac{3}{2}} - 3x \right) dx$. *(3 marks)*

(d) Calculate the area of the shaded region bounded by the curve and the tangents OP and AP. *(5 marks)*

END OF QUESTIONS

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General Certificate of Education
June 2006
Advanced Subsidiary Examination

MATHEMATICS **MPC2**
Unit Pure Core 2

Monday 22 May 2006 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the **blue** AQA booklet of formulae and statistical tables

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

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- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
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- The marks for questions are shown in brackets.

Advice
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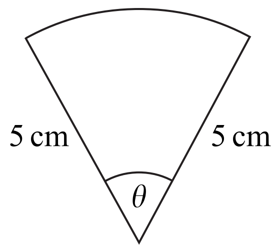
Answer **all** questions.

1 The diagram shows a sector of a circle of radius 5 cm and angle θ radians.

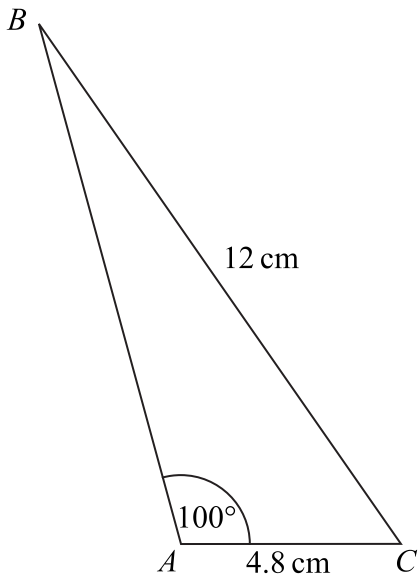


The area of the sector is $8.1\ \text{cm}^2$.

(a) Show that $\theta = 0.648$. *(2 marks)*

(b) Find the perimeter of the sector. *(3 marks)*

2 The diagram shows a triangle *ABC*.



The lengths of *AC* and *BC* are 4.8 cm and 12 cm respectively.

The size of the angle *BAC* is $100°$.

(a) Show that angle $ABC = 23.2°$, correct to the nearest $0.1°$. *(3 marks)*

(b) Calculate the area of triangle *ABC*, giving your answer in cm^2 to three significant figures. *(3 marks)*

3 The first term of an arithmetic series is 1. The common difference of the series is 6.

(a) Find the tenth term of the series. *(2 marks)*

(b) The sum of the first n terms of the series is 7400.

(i) Show that $3n^2 - 2n - 7400 = 0$. *(3 marks)*

(ii) Find the value of n. *(2 marks)*

4 (a) The expression $(1 - 2x)^4$ can be written in the form

$$1 + px + qx^2 - 32x^3 + 16x^4$$

By using the binomial expansion, or otherwise, find the values of the integers p and q. *(3 marks)*

(b) Find the coefficient of x in the expansion of $(2 + x)^9$. *(2 marks)*

(c) Find the coefficient of x in the expansion of $(1 - 2x)^4(2 + x)^9$. *(3 marks)*

5 (a) Given that

$$\log_a x = 2\log_a 6 - \log_a 3$$

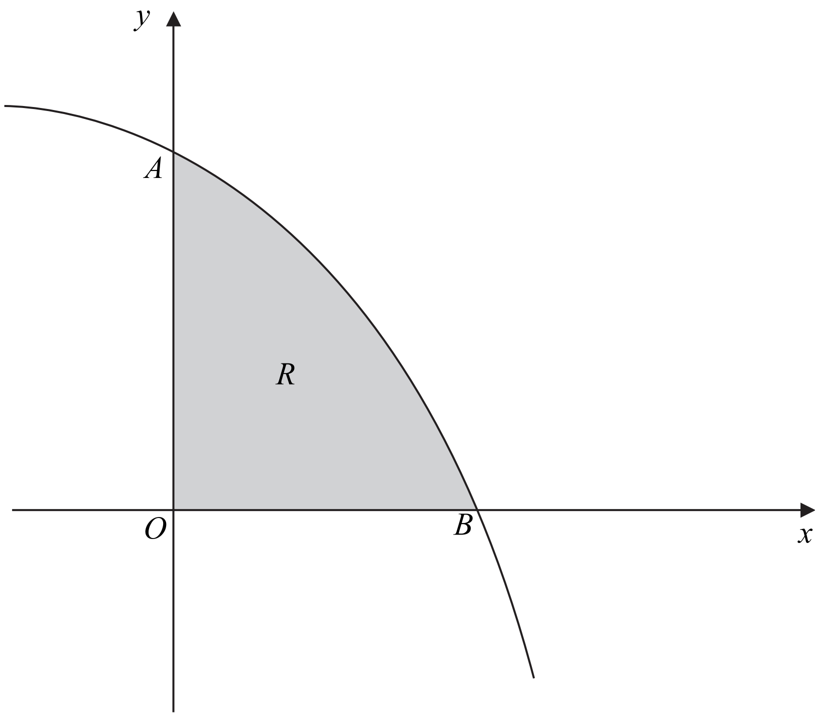
show that $x = 12$. *(3 marks)*

(b) Given that

$$\log_a y + \log_a 5 = 7$$

express y in terms of a, giving your answer in a form not involving logarithms. *(3 marks)*

6 The diagram shows a sketch of the curve with equation $y = 27 - 3^x$.



The curve $y = 27 - 3^x$ intersects the y-axis at the point A and the x-axis at the point B.

(a) (i) Find the y-coordinate of point A. *(2 marks)*

(ii) Verify that the x-coordinate of point B is 3. *(1 mark)*

(b) The region, R, bounded by the curve $y = 27 - 3^x$ and the coordinate axes is shaded. Use the trapezium rule with four ordinates (three strips) to find an approximate value for the area of R. *(4 marks)*

(c) (i) Use logarithms to solve the equation $3^x = 13$, giving your answer to four decimal places. *(3 marks)*

(ii) The line $y = k$ intersects the curve $y = 27 - 3^x$ at the point where $3^x = 13$. Find the value of k. *(1 mark)*

(d) (i) Describe the single geometrical transformation by which the curve with equation $y = -3^x$ can be obtained **from** the curve $y = 27 - 3^x$. *(2 marks)*

(ii) Sketch the curve $y = -3^x$. *(2 marks)*

7 At the point (x, y), where $x > 0$, the gradient of a curve is given by

$$\frac{dy}{dx} = 3x^{\frac{1}{2}} + \frac{16}{x^2} - 7$$

(a) (i) Verify that $\frac{dy}{dx} = 0$ when $x = 4$. *(1 mark)*

(ii) Write $\frac{16}{x^2}$ in the form $16x^k$, where k is an integer. *(1 mark)*

(iii) Find $\frac{d^2y}{dx^2}$. *(3 marks)*

(iv) Hence determine whether the point where $x = 4$ is a maximum or a minimum, giving a reason for your answer. *(2 marks)*

(b) The point $P(1, 8)$ lies on the curve.

(i) Show that the gradient of the curve at the point P is 12. *(1 mark)*

(ii) Find an equation of the normal to the curve at P. *(3 marks)*

(c) (i) Find $\int (3x^{\frac{1}{2}} + \frac{16}{x^2} - 7)\, dx$. *(3 marks)*

(ii) Hence find the equation of the curve which passes through the point $P(1, 8)$. *(3 marks)*

8 (a) Describe the single geometrical transformation by which the curve with equation $y = \tan\frac{1}{2}x$ can be obtained from the curve $y = \tan x$. *(2 marks)*

(b) Solve the equation $\tan\frac{1}{2}x = 3$ in the interval $\mathbf{0 < x < 4\pi}$, giving your answers in radians to three significant figures. *(4 marks)*

(c) Solve the equation

$$\cos\theta(\sin\theta - 3\cos\theta) = 0$$

in the interval $0 < \theta < 2\pi$, giving your answers in radians to three significant figures. *(5 marks)*

END OF QUESTIONS

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General Certificate of Education
January 2007
Advanced Subsidiary Examination

MATHEMATICS **MPC2**
Unit Pure Core 2

Wednesday 10 January 2007 1.30 pm to 3.00 pm

For this paper you must have:
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You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
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- Answer **all** questions.
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Information
- The maximum mark for this paper is 75.
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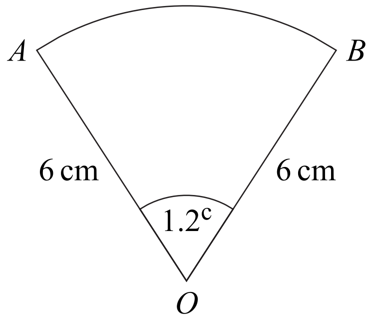
Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The diagram shows a sector OAB of a circle with centre O.



The radius of the circle is 6 cm and the angle AOB is 1.2 radians.

(a) Find the area of the sector OAB. *(2 marks)*

(b) Find the perimeter of the sector OAB. *(3 marks)*

2 Use the trapezium rule with four ordinates (three strips) to find an approximate value for

$$\int_0^3 \sqrt{2^x}\, dx$$

giving your answer to three decimal places. *(4 marks)*

3 (a) Write down the values of p, q and r given that:

(i) $64 = 8^p$;

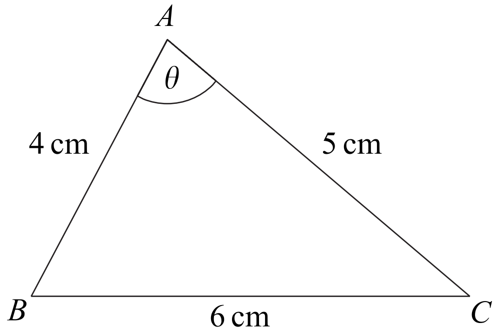
(ii) $\frac{1}{64} = 8^q$;

(iii) $\sqrt{8} = 8^r$. *(3 marks)*

(b) Find the value of x for which

$$\frac{8^x}{\sqrt{8}} = \frac{1}{64}$$ *(2 marks)*

4 The triangle ABC, shown in the diagram, is such that $BC = 6$ cm, $AC = 5$ cm and $AB = 4$ cm. The angle BAC is θ.



(a) Use the cosine rule to show that $\cos\theta = \frac{1}{8}$. *(3 marks)*

(b) Hence use a trigonometrical identity to show that $\sin\theta = \frac{3\sqrt{7}}{8}$. *(3 marks)*

(c) Hence find the area of the triangle ABC. *(2 marks)*

5 The second term of a geometric series is 48 and the fourth term is 3.

(a) Show that one possible value for the common ratio, r, of the series is $-\frac{1}{4}$ and state the other value. *(4 marks)*

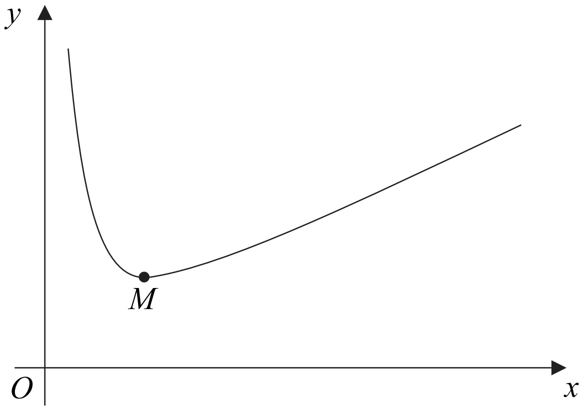
(b) In the case when $r = -\frac{1}{4}$, find:

(i) the first term; *(1 mark)*

(ii) the sum to infinity of the series. *(2 marks)*

Turn over ▶

6 A curve C is defined for $x > 0$ by the equation $y = x + 1 + \dfrac{4}{x^2}$ and is sketched below.



(a) (i) Given that $y = x + 1 + \dfrac{4}{x^2}$, find $\dfrac{dy}{dx}$. *(3 marks)*

(ii) The curve C has a minimum point M. Find the coordinates of M. *(4 marks)*

(iii) Find an equation of the normal to C at the point $(1, 6)$. *(4 marks)*

(b) (i) Find $\displaystyle\int \left(x + 1 + \frac{4}{x^2} \right) dx$. *(3 marks)*

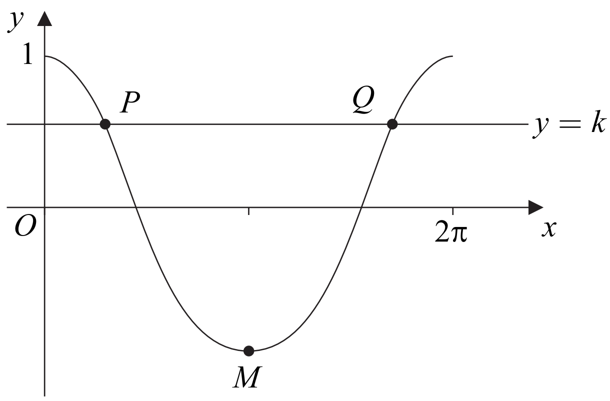
(ii) Hence find the area of the region bounded by the curve C, the lines $x = 1$ and $x = 4$ and the x-axis. *(2 marks)*

7 (a) The first four terms of the binomial expansion of $(1 + 2x)^8$ in ascending powers of x are $1 + ax + bx^2 + cx^3$. Find the values of the integers a, b and c. *(4 marks)*

(b) Hence find the coefficient of x^3 in the expansion of $\left(1 + \frac{1}{2}x\right)(1 + 2x)^8$. *(3 marks)*

8 (a) Solve the equation $\cos x = 0.3$ in the interval $0 \leqslant x \leqslant 2\pi$, giving your answers in radians to three significant figures. *(3 marks)*

(b) The diagram shows the graph of $y = \cos x$ for $0 \leqslant x \leqslant 2\pi$ and the line $y = k$.



The line $y = k$ intersects the curve $y = \cos x$, $0 \leqslant x \leqslant 2\pi$, at the points P and Q. The point M is the minimum point of the curve.

(i) Write down the coordinates of the point M. *(2 marks)*

(ii) The x-coordinate of P is α.

Write down the x-coordinate of Q in terms of π and α. *(1 mark)*

(c) Describe the geometrical transformation that maps the graph of $y = \cos x$ onto the graph of $y = \cos 2x$. *(2 marks)*

(d) Solve the equation $\cos 2x = \cos \frac{4\pi}{5}$ in the interval $0 \leqslant x \leqslant 2\pi$, giving the values of x in terms of π. *(4 marks)*

Turn over for the next question

9 (a) Solve the equation $3\log_a x = \log_a 8$. *(2 marks)*

(b) Show that

$$3\log_a 6 - \log_a 8 = \log_a 27$$ *(3 marks)*

(c) (i) The point $P(3, p)$ lies on the curve $y = 3\log_{10} x - \log_{10} 8$.

Show that $p = \log_{10}\left(\frac{27}{8}\right)$. *(2 marks)*

(ii) The point $Q(6, q)$ also lies on the curve $y = 3\log_{10} x - \log_{10} 8$.

Show that the gradient of the line PQ is $\log_{10} 2$. *(4 marks)*

END OF QUESTIONS

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General Certificate of Education
June 2007
Advanced Subsidiary Examination

MATHEMATICS **MPC2**
Unit Pure Core 2

Monday 21 May 2007 9.00 am to 10.30 am

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Time allowed: 1 hour 30 minutes

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Advice
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Answer **all** questions.

1 (a) Simplify:

(i) $x^{\frac{3}{2}} \times x^{\frac{1}{2}}$; *(1 mark)*

(ii) $x^{\frac{3}{2}} \div x$; *(1 mark)*

(iii) $\left(x^{\frac{3}{2}}\right)^2$. *(1 mark)*

(b) (i) Find $\int 3x^{\frac{1}{2}}\,dx$. *(3 marks)*

(ii) Hence find the value of $\int_1^9 3x^{\frac{1}{2}}\,dx$. *(2 marks)*

2 The nth term of a geometric sequence is u_n, where

$$u_n = 3 \times 4^n$$

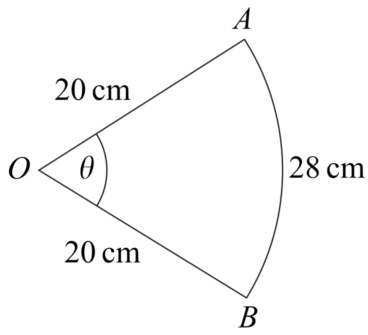
(a) Find the value of u_1 and show that $u_2 = 48$. *(2 marks)*

(b) Write down the common ratio of the geometric sequence. *(1 mark)*

(c) (i) Show that the sum of the first 12 terms of the geometric sequence is $4^k - 4$, where k is an integer. *(3 marks)*

(ii) Hence find the value of $\sum_{n=2}^{12} u_n$. *(1 mark)*

3 The diagram shows a sector OAB of a circle with centre O and radius 20 cm. The angle between the radii OA and OB is θ radians.

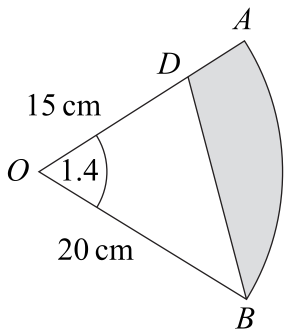


The length of the arc AB is 28 cm.

(a) Show that $\theta = 1.4$. *(2 marks)*

(b) Find the area of the sector OAB. *(2 marks)*

(c) The point D lies on OA. The region bounded by the line BD, the line DA and the arc AB is shaded.



The length of OD is 15 cm.

(i) Find the area of the shaded region, giving your answer to three significant figures. *(3 marks)*

(ii) Use the cosine rule to calculate the length of BD, giving your answer to three significant figures. *(3 marks)*

Turn over ▶

4 An arithmetic series has first term a and common difference d.

The sum of the first 29 terms is 1102.

(a) Show that $a + 14d = 38$. *(3 marks)*

(b) The sum of the second term and the seventh term is 13.

Find the value of a and the value of d. *(4 marks)*

5 A curve is defined for $x > 0$ by the equation

$$y = \left(1 + \frac{2}{x}\right)^2$$

The point P lies on the curve where $x = 2$.

(a) Find the y-coordinate of P. *(1 mark)*

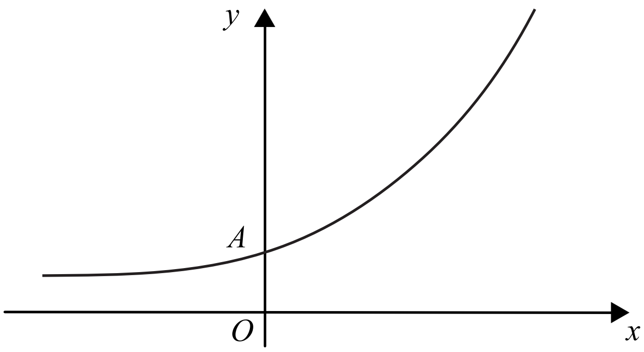
(b) Expand $\left(1 + \frac{2}{x}\right)^2$. *(2 marks)*

(c) Find $\frac{dy}{dx}$. *(3 marks)*

(d) Hence show that the gradient of the curve at P is -2. *(2 marks)*

(e) Find the equation of the normal to the curve at P, giving your answer in the form $x + by + c = 0$, where b and c are integers. *(4 marks)*

6 The diagram shows a sketch of the curve with equation $y = 3(2^x + 1)$.



The curve $y = 3(2^x + 1)$ intersects the y-axis at the point A.

(a) Find the y-coordinate of the point A. *(2 marks)*

(b) Use the trapezium rule with four ordinates (three strips) to find an approximate value for $\int_0^6 3(2^x + 1)\,dx$. *(4 marks)*

(c) The line $y = 21$ intersects the curve $y = 3(2^x + 1)$ at the point P.

(i) Show that the x-coordinate of P satisfies the equation

$$2^x = 6$$ *(1 mark)*

(ii) Use logarithms to find the x-coordinate of P, giving your answer to three significant figures. *(3 marks)*

Turn over for the next question

7 (a) Sketch the graph of $y = \tan x$ for $0° \leqslant x \leqslant 360°$. *(3 marks)*

(b) Write down the **two** solutions of the equation $\tan x = \tan 61°$ in the interval $0° \leqslant x \leqslant 360°$. *(2 marks)*

(c) (i) Given that $\sin\theta + \cos\theta = 0$, show that $\tan\theta = -1$. *(1 mark)*

(ii) Hence solve the equation $\sin(x - 20°) + \cos(x - 20°) = 0$ in the interval $0° \leqslant x \leqslant 360°$. *(4 marks)*

(d) Describe the single geometrical transformation that maps the graph of $y = \tan x$ onto the graph of $y = \tan(x - 20°)$. *(2 marks)*

(e) The curve $y = \tan x$ is stretched in the x-direction with scale factor $\frac{1}{4}$ to give the curve with equation $y = \mathrm{f}(x)$. Write down an expression for $\mathrm{f}(x)$. *(1 mark)*

8 (a) It is given that n satisfies the equation

$$\log_a n = \log_a 3 + \log_a(2n - 1)$$

Find the value of n. *(3 marks)*

(b) Given that $\log_a x = 3$ and $\log_a y - 3\log_a 2 = 4$:

(i) express x in terms of a; *(1 mark)*

(ii) express xy in terms of a. *(4 marks)*

END OF QUESTIONS

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General Certificate of Education
January 2008
Advanced Subsidiary Examination

MATHEMATICS **MPC2**
Unit Pure Core 2

Wednesday 9 January 2008 1.30 pm to 3.00 pm

For this paper you must have:
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Advice
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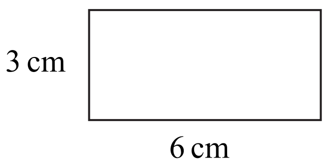
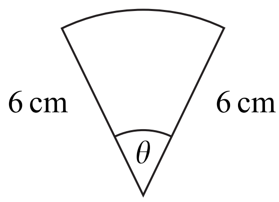
Answer **all** questions.

1 The diagrams show a rectangle of length 6 cm and width 3 cm, and a sector of a circle of radius 6 cm and angle θ radians.





The area of the rectangle is twice the area of the sector.

(a) Show that $\theta = 0.5$. *(3 marks)*

(b) Find the perimeter of the sector. *(3 marks)*

2 The arithmetic series

$$51 + 58 + 65 + 72 + \ldots + 1444$$

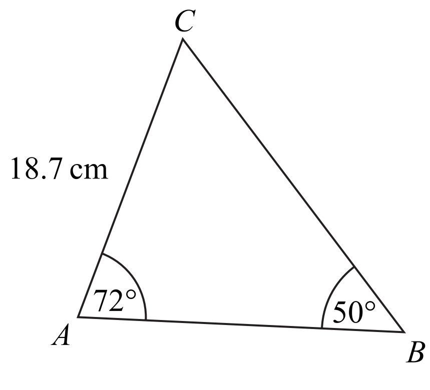
has 200 terms.

(a) Write down the common difference of the series. *(1 mark)*

(b) Find the 101st term of the series. *(2 marks)*

(c) Find the sum of **the last** 100 terms of the series. *(2 marks)*

3 The diagram shows a triangle *ABC*. The length of *AC* is 18.7 cm, and the sizes of angles *BAC* and *ABC* are 72° and 50° respectively.



(a) Show that the length of $BC = 23.2$ cm, correct to the nearest 0.1 cm. *(3 marks)*

(b) Calculate the area of triangle *ABC*, giving your answer to the nearest cm^2. *(3 marks)*

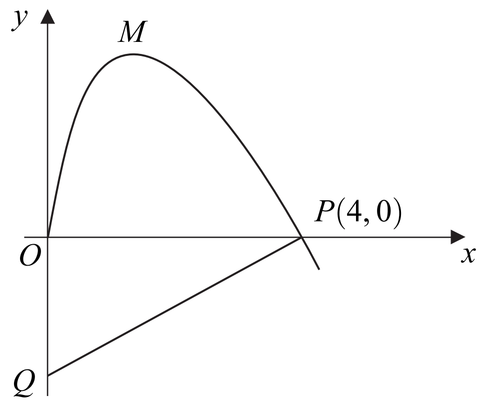
4 Use the trapezium rule with four ordinates (three strips) to find an approximate value for

$$\int_0^3 \sqrt{x^2+3}\, dx$$

giving your answer to three decimal places. *(4 marks)*

5 A curve, drawn from the origin O, crosses the x-axis at the point $P(4, 0)$.

The normal to the curve at P meets the y-axis at the point Q, as shown in the diagram.



The curve, defined for $x \geqslant 0$, has equation

$$y = 4x^{\frac{1}{2}} - x^{\frac{3}{2}}$$

(a) (i) Find $\dfrac{dy}{dx}$. *(3 marks)*

(ii) Show that the gradient of the curve at $P(4, 0)$ is -2. *(2 marks)*

(iii) Find an equation of the normal to the curve at $P(4, 0)$. *(3 marks)*

(iv) Find the y-coordinate of Q and hence find the area of triangle OPQ. *(3 marks)*

(v) The curve has a maximum point M. Find the x-coordinate of M. *(3 marks)*

(b) (i) Find $\displaystyle\int \left(4x^{\frac{1}{2}} - x^{\frac{3}{2}}\right) dx$. *(3 marks)*

(ii) Find the total area of the region bounded by the curve and the lines PQ and QO. *(3 marks)*

Turn over ▶

6 (a) Using the binomial expansion, or otherwise:

(i) express $(1 + x)^3$ in ascending powers of x; *(2 marks)*

(ii) express $(1 + x)^4$ in ascending powers of x. *(2 marks)*

(b) Hence, or otherwise:

(i) express $(1 + 4x)^3$ in ascending powers of x; *(2 marks)*

(ii) express $(1 + 3x)^4$ in ascending powers of x. *(2 marks)*

(c) Show that the expansion of

$$(1 + 3x)^4 - (1 + 4x)^3$$

can be written in the form

$$px^2 + qx^3 + rx^4$$

where p, q and r are integers. *(2 marks)*

7 (a) Given that

$$\log_a x = \log_a 16 - \log_a 2$$

write down the value of x. *(1 mark)*

(b) Given that

$$\log_a y = 2 \log_a 3 + \log_a 4 + 1$$

express y in terms of a, giving your answer in a form **not** involving logarithms. *(3 marks)*

8 (a) Sketch the graph of $y = 3^x$, stating the coordinates of the point where the graph crosses the y-axis. *(2 marks)*

(b) Describe a single geometrical transformation that maps the graph of $y = 3^x$:

(i) onto the graph of $y = 3^{2x}$; *(2 marks)*

(ii) onto the graph of $y = 3^{x+1}$. *(2 marks)*

(c) (i) Using the substitution $Y = 3^x$, show that the equation

$$9^x - 3^{x+1} + 2 = 0$$

can be written as

$$(Y - 1)(Y - 2) = 0$$ *(2 marks)*

(ii) Hence show that the equation $9^x - 3^{x+1} + 2 = 0$ has a solution $x = 0$ and, by using logarithms, find the other solution, giving your answer to four decimal places. *(4 marks)*

9 (a) Given that

$$\frac{3 + \sin^2 \theta}{\cos \theta - 2} = 3 \cos \theta$$

show that

$$\cos \theta = -\frac{1}{2}$$ *(4 marks)*

(b) Hence solve the equation

$$\frac{3 + \sin^2 3x}{\cos 3x - 2} = 3 \cos 3x$$

giving all solutions in degrees in the interval $0° < x < 180°$. *(4 marks)*

END OF QUESTIONS

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General Certificate of Education
June 2008
Advanced Subsidiary Examination

ASSESSMENT and QUALIFICATIONS ALLIANCE

MATHEMATICS **MPC2**
Unit Pure Core 2

Thursday 15 May 2008 9.00 am to 10.30 am

For this paper you must have:
• an 8-page answer book
• the blue AQA booklet of formulae and statistical tables.
You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 (a) Write $\sqrt{x^3}$ in the form x^k, where k is a fraction. *(1 mark)*

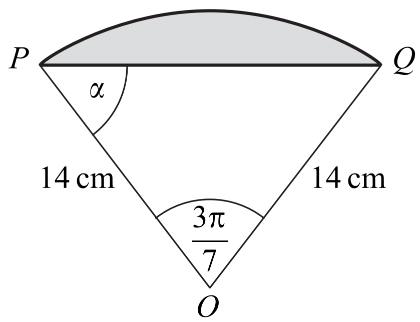
(b) A curve, defined for $x \geqslant 0$, has equation

$$y = x^2 - \sqrt{x^3}$$

(i) Find $\frac{dy}{dx}$. *(3 marks)*

(ii) Find the equation of the tangent to the curve at the point where $x = 4$, giving your answer in the form $y = mx + c$. *(5 marks)*

2 The diagram shows a shaded segment of a circle with centre O and radius 14 cm, where PQ is a chord of the circle.



In triangle OPQ, angle $POQ = \frac{3\pi}{7}$ radians and angle $OPQ = \alpha$ radians.

(a) Find the length of the arc PQ, giving your answer as a multiple of π. *(2 marks)*

(b) Find α in terms of π. *(2 marks)*

(c) Find the **perimeter** of the shaded segment, giving your answer to three significant figures. *(2 marks)*

3 A geometric series begins

$$20 + 16 + 12.8 + 10.24 + ...$$

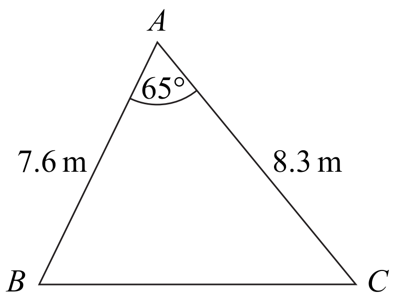
(a) Find the common ratio of the series. *(1 mark)*

(b) Find the sum to infinity of the series. *(2 marks)*

(c) Find the sum of the first 20 terms of the series, giving your answer to three decimal places. *(2 marks)*

(d) Prove that the nth term of the series is 25×0.8^n. *(2 marks)*

4 The diagram shows a triangle ABC.



The size of angle BAC is 65°, and the lengths of AB and AC are 7.6 m and 8.3 m respectively.

(a) Show that the length of BC is 8.56 m, correct to three significant figures. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer in m^2 to three significant figures. *(2 marks)*

(c) The perpendicular from A to BC meets BC at the point D.

Calculate the length of AD, giving your answer to the nearest 0.1 m. *(3 marks)*

5 (a) Write down the value of:

(i) $\log_a 1$; *(1 mark)*

(ii) $\log_a a$. *(1 mark)*

(b) Given that

$$\log_a x = \log_a 5 + \log_a 6 - \log_a 1.5$$

find the value of x. *(3 marks)*

Turn over ▶

6 The nth term of a sequence is u_n.

The sequence is defined by

$$u_{n+1} = pu_n + q$$

where p and q are constants.

The first three terms of the sequence are given by

$$u_1 = -8 \qquad u_2 = 8 \qquad u_3 = 4$$

(a) Show that $q = 6$ and find the value of p. *(5 marks)*

(b) Find the value of u_4. *(1 mark)*

(c) The limit of u_n as n tends to infinity is L.

(i) Write down an equation for L. *(1 mark)*

(ii) Hence find the value of L. *(2 marks)*

7 (a) The expression $\left(1 + \frac{4}{x^2}\right)^3$ can be written in the form

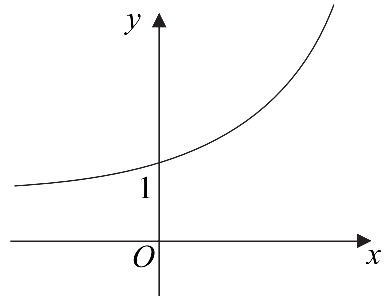
$$1 + \frac{p}{x^2} + \frac{q}{x^4} + \frac{64}{x^6}$$

By using the binomial expansion, or otherwise, find the values of the integers p and q. *(3 marks)*

(b) (i) Hence find $\int \left(1 + \frac{4}{x^2}\right)^3 \mathrm{d}x$. *(4 marks)*

(ii) Hence find the value of $\int_1^2 \left(1 + \frac{4}{x^2}\right)^3 \mathrm{d}x$. *(2 marks)*

8 The diagram shows a sketch of the curve with equation $y = 6^x$.



(a) (i) Use the trapezium rule with five ordinates (four strips) to find an approximate value for $\int_0^2 6^x \, dx$, giving your answer to three significant figures. *(4 marks)*

(ii) Explain, with the aid of a diagram, whether your approximate value will be an overestimate or an underestimate of the true value of $\int_0^2 6^x \, dx$. *(2 marks)*

(b) (i) Describe a single geometrical transformation that maps the graph of $y = 6^x$ onto the graph of $y = 6^{3x}$. *(2 marks)*

(ii) The line $y = 84$ intersects the curve $y = 6^{3x}$ at the point A. By using logarithms, find the x-coordinate of A, giving your answer to three decimal places. *(4 marks)*

(c) The graph of $y = 6^x$ is translated by $\begin{bmatrix} 1 \\ -2 \end{bmatrix}$ to give the graph of the curve with equation $y = f(x)$. Write down an expression for $f(x)$. *(2 marks)*

9 (a) Solve the equation $\sin 2x = \sin 48°$, giving the values of x in the interval $0° \leqslant x < 360°$. *(4 marks)*

(b) Solve the equation $2\sin\theta - 3\cos\theta = 0$ in the interval $0° \leqslant \theta < 360°$, giving your answers to the nearest 0.1°. *(4 marks)*

END OF QUESTIONS

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General Certificate of Education
January 2009
Advanced Subsidiary Examination

MATHEMATICS **MPC2**
Unit Pure Core 2

Tuesday 13 January 2009 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

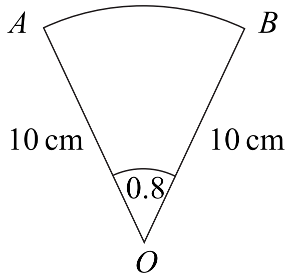
Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The diagram shows a sector OAB of a circle with centre O and radius 10 cm.



The angle AOB is 0.8 radians.

(a) Find the area of the sector. *(2 marks)*

(b) (i) Find the perimeter of the sector OAB. *(3 marks)*

(ii) The perimeter of the sector OAB is equal to the perimeter of a square. Find the area of the square. *(2 marks)*

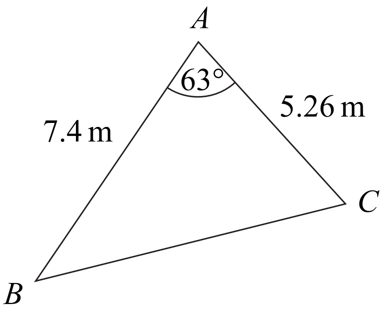
2 (a) Use the trapezium rule with four ordinates (three strips) to find an approximate value for

$$\int_{1.5}^{6} x^2\sqrt{x^2-1}\,\mathrm{d}x$$

giving your answer to three significant figures. *(4 marks)*

(b) State how you could obtain a better approximation to the value of the integral using the trapezium rule. *(1 mark)*

3 The diagram shows a triangle ABC.



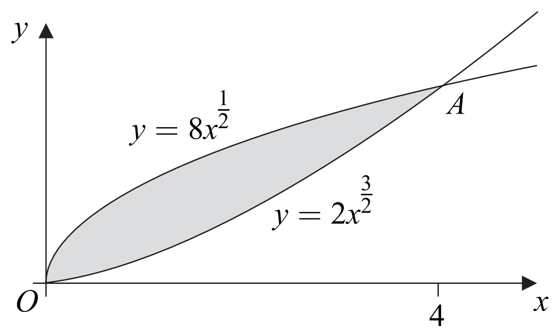
The size of angle A is 63°, and the lengths of AB and AC are 7.4 m and 5.26 m respectively.

(a) Calculate the area of triangle ABC, giving your answer in m^2 to three significant figures. *(2 marks)*

(b) Show that the length of BC is 6.86 m, correct to three significant figures. *(3 marks)*

(c) Find the value of **sin *B*** to two significant figures. *(2 marks)*

4 The diagram shows a sketch of the curves with equations $y = 2x^{\frac{3}{2}}$ and $y = 8x^{\frac{1}{2}}$.



The curves intersect at the origin and at the point A, where $x = 4$.

(a) (i) For the curve $y = 2x^{\frac{3}{2}}$, find the value of $\dfrac{dy}{dx}$ when $x = 4$. *(2 marks)*

(ii) Find an equation of the normal to the curve $y = 2x^{\frac{3}{2}}$ at the point A. *(4 marks)*

(b) (i) Find $\int 8x^{\frac{1}{2}}\,dx$. *(2 marks)*

(ii) Find the area of the shaded region bounded by the two curves. *(4 marks)*

(c) Describe a single geometrical transformation that maps the graph of $y = 2x^{\frac{3}{2}}$ onto the graph of $y = 2(x+3)^{\frac{3}{2}}$. *(2 marks)*

Turn over ▶

5 (a) By using the binomial expansion, or otherwise, express $(1+2x)^4$ in the form

$$1 + ax + bx^2 + cx^3 + 16x^4$$

where a, b and c are integers. *(4 marks)*

(b) Hence show that $(1+2x)^4 + (1-2x)^4 = 2 + 48x^2 + 32x^4$. *(3 marks)*

(c) Hence show that the curve with equation

$$y = (1+2x)^4 + (1-2x)^4$$

has just one stationary point and state its coordinates. *(4 marks)*

6 (a) Write each of the following in the form $\log_a k$, where k is an integer:

(i) $\log_a 4 + \log_a 10$; *(1 mark)*

(ii) $\log_a 16 - \log_a 2$; *(1 mark)*

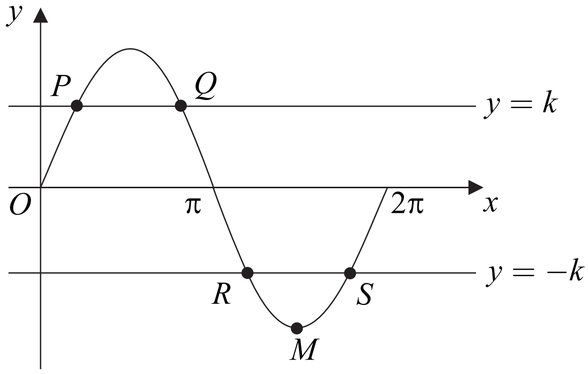
(iii) $3\log_a 5$. *(1 mark)*

(b) Use logarithms to solve the equation $(1.5)^{3x} = 7.5$, giving your value of x to three decimal places. *(3 marks)*

(c) Given that $\log_2 p = m$ and $\log_8 q = n$, express pq in the form 2^y, where y is an expression in m and n. *(3 marks)*

7 (a) Solve the equation $\sin x = 0.8$ in the interval $0 \leqslant x \leqslant 2\pi$, giving your answers in radians to three significant figures. *(3 marks)*

(b) The diagram shows the graph of the curve $y = \sin x$, $0 \leqslant x \leqslant 2\pi$ and the lines $y = k$ and $y = -k$.



The line $y = k$ intersects the curve at the points P and Q, and the line $y = -k$ intersects the curve at the points R and S.

The point M is the minimum point of the curve.

(i) Write down the coordinates of the point M. *(2 marks)*

(ii) The x-coordinate of P is α.

Write down the x-coordinate of the point Q in terms of π and α. *(1 mark)*

(iii) Find the length of RS in terms of π and α, giving your answer in its simplest form. *(2 marks)*

(c) Sketch the graph of $y = \sin 2x$ for $0 \leqslant x \leqslant 2\pi$, indicating the coordinates of points where the graph intersects the x-axis and the coordinates of any maximum points. *(5 marks)*

8 The 25th term of an arithmetic series is 38.

The sum of the first 40 terms of the series is 1250.

(a) Show that the common difference of this series is 1.5. *(6 marks)*

(b) Find the number of terms in the series which are less than 100. *(3 marks)*

END OF QUESTIONS

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Centre Number						Candidate Number				
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Other Names										
Candidate Signature										

For Examiner's Use	
Examiner's Initials	
Question	Mark
1	
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TOTAL	

AQA

General Certificate of Education
Advanced Subsidiary Examination
June 2009

Mathematics MPC2

Unit Pure Core 2

Specimen paper for examinations in June 2010 onwards

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the space provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

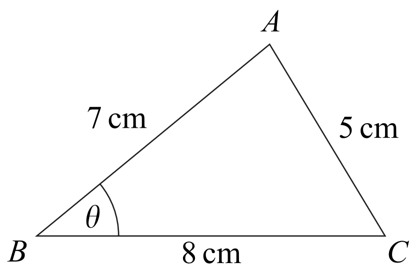
Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Do not write outside the box

Answer **all** questions in the spaces provided.

1 The triangle ABC, shown in the diagram, is such that $AB = 7\text{ cm}$, $AC = 5\text{ cm}$, $BC = 8\text{ cm}$ and angle $ABC = \theta$.



(a) Show that $\theta = 38.2°$, correct to the nearest $0.1°$. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer, in cm^2, to three significant figures. *(2 marks)*

QUESTION PART REFERENCE

Do not write outside the box

2 (a) Write down the value of n given that $\frac{1}{x^4} = x^n$. *(1 mark)*

(b) Expand $\left(1 + \frac{3}{x^2}\right)^2$. *(2 marks)*

(c) Hence find $\int \left(1 + \frac{3}{x^2}\right)^2 \mathrm{d}x$. *(3 marks)*

(d) Hence find the exact value of $\int_1^3 \left(1 + \frac{3}{x^2}\right)^2 \mathrm{d}x$. *(2 marks)*

QUESTION PART REFERENCE

3 The nth term of a sequence is u_n.

The sequence is defined by

$$u_{n+1} = ku_n + 12$$

where k is a constant.

The first two terms of the sequence are given by

$$u_1 = 16 \qquad u_2 = 24$$

(a) Show that $k = 0.75$. *(2 marks)*

(b) Find the value of u_3 and the value of u_4. *(2 marks)*

(c) The limit of u_n as n tends to infinity is L.

(i) Write down an equation for L. *(1 mark)*

(ii) Hence find the value of L. *(2 marks)*

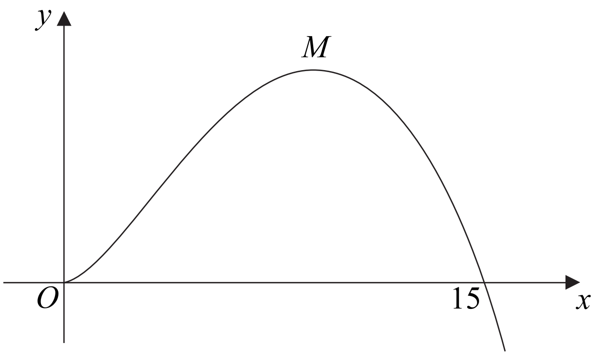
QUESTION PART REFERENCE

4 (a) Use the trapezium rule with four ordinates (three strips) to find an approximate value for $\int_0^6 \sqrt{x^3 + 1}\,dx$, giving your answer to four significant figures. *(4 marks)*

(b) The curve with equation $y = \sqrt{x^3 + 1}$ is stretched parallel to the x-axis with scale factor $\frac{1}{2}$ to give the curve with equation $y = f(x)$. Write down an expression for $f(x)$. *(2 marks)*

QUESTION PART REFERENCE

5 The diagram shows part of a curve with a maximum point M.



The equation of the curve is

$$y = 15x^{\frac{3}{2}} - x^{\frac{5}{2}}$$

(a) Find $\dfrac{dy}{dx}$. *(3 marks)*

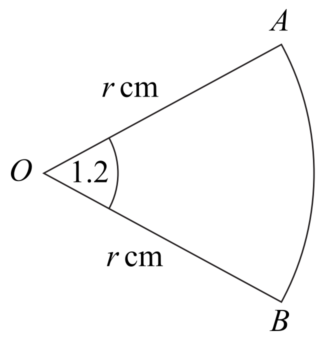
(b) Hence find the coordinates of the maximum point M. *(4 marks)*

(c) The point $P(1, 14)$ lies on the curve. Show that the equation of the tangent to the curve at P is $y = 20x - 6$. *(3 marks)*

(d) The tangents to the curve at the points P and M intersect at the point R. Find the length of RM. *(3 marks)*

QUESTION PART REFERENCE

6 The diagram shows a sector OAB of a circle with centre O and radius r cm.



The angle AOB is 1.2 radians. The area of the sector is $33.75\,\text{cm}^2$.

Find the perimeter of the sector. *(6 marks)*

QUESTION PART REFERENCE

7 A geometric series has second term 375 and fifth term 81.

(a) (i) Show that the common ratio of the series is 0.6. *(3 marks)*

(ii) Find the first term of the series. *(2 marks)*

(b) Find the sum to infinity of the series. *(2 marks)*

(c) The nth term of the series is u_n. Find the value of $\sum_{n=6}^{\infty} u_n$. *(4 marks)*

QUESTION PART REFERENCE

8 (a) Given that $\dfrac{\sin\theta - \cos\theta}{\cos\theta} = 4$, prove that $\tan\theta = 5$. *(2 marks)*

(b) (i) Use an appropriate identity to show that the equation

$$2\cos^2 x - \sin x = 1$$

can be written as

$$2\sin^2 x + \sin x - 1 = 0$$ *(2 marks)*

(ii) Hence solve the equation

$$2\cos^2 x - \sin x = 1$$

giving all solutions in the interval $0° \leqslant x \leqslant 360°$. *(5 marks)*

QUESTION PART REFERENCE

9 (a) (i) Find the value of p for which $\sqrt{125} = 5^p$. *(2 marks)*

(ii) Hence solve the equation $5^{2x} = \sqrt{125}$. *(1 mark)*

(b) Use logarithms to solve the equation $3^{2x-1} = 0.05$, giving your value of x to four decimal places. *(3 marks)*

(c) It is given that

$$\log_a x = 2(\log_a 3 + \log_a 2) - 1$$

Express x in terms of a, giving your answer in a form not involving logarithms. *(4 marks)*

QUESTION PART REFERENCE

General Certificate of Education
Advanced Subsidiary Examination
January 2010

Mathematics MPC2

Unit Pure Core 2

Monday 11 January 2010 9.00 am to 10.30 am

For this paper you must have:
- an 8-page answer book
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

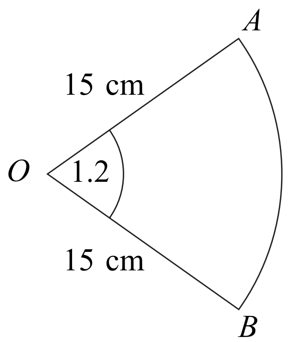
Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The **Examining Body** for this paper is AQA. The **Paper Reference** is MPC2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions.

1 The diagram shows a sector OAB of a circle with centre O.

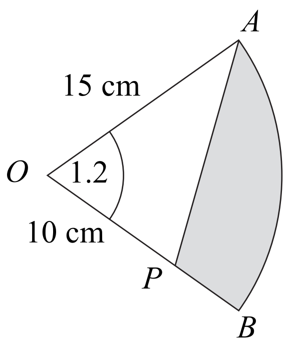


The radius of the circle is 15 cm and angle $AOB = 1.2$ radians.

(a) (i) Show that the area of the sector is 135 cm^2. *(2 marks)*

(ii) Calculate the length of the arc AB. *(2 marks)*

(b) The point P lies on the radius OB such that $OP = 10$ cm, as shown in the diagram below.



Calculate the perimeter of the shaded region bounded by AP, PB and the arc AB, giving your answer to three significant figures. *(5 marks)*

2 At the point (x, y) on a curve, where $x > 0$, the gradient is given by

$$\frac{dy}{dx} = 7\sqrt{x^5} - 4$$

(a) Write $\sqrt{x^5}$ in the form x^k, where k is a fraction. *(1 mark)*

(b) Find $\int \left(7\sqrt{x^5} - 4\right) dx$. *(3 marks)*

(c) Hence find the equation of the curve, given that the curve passes through the point $(1, 3)$. *(3 marks)*

3 (a) Find the value of x in each of the following:

(i) $\log_9 x = 0$; *(1 mark)*

(ii) $\log_9 x = \frac{1}{2}$. *(1 mark)*

(b) Given that

$$2\log_a n = \log_a 18 + \log_a(n - 4)$$

find the possible values of n. *(5 marks)*

4 An arithmetic series has first term a and common difference d.

The sum of the first 31 terms of the series is 310.

(a) Show that $a + 15d = 10$. *(3 marks)*

(b) Given also that the 21st term is twice the 16th term, find the value of d. *(3 marks)*

(c) The nth term of the series is u_n. Given that $\sum_{n=1}^{k} u_n = 0$, find the value of k. *(4 marks)*

5 A curve has equation $y = \dfrac{1}{x^3} + 48x$.

(a) Find $\dfrac{dy}{dx}$. *(3 marks)*

(b) Hence find the equation of each of the two tangents to the curve that are parallel to the x-axis. *(4 marks)*

(c) Find an equation of the normal to the curve at the point $(1, 49)$. *(3 marks)*

6 (a) Sketch the curve with equation $y = 2^x$, indicating the coordinates of any point where the curve intersects the coordinate axes. *(2 marks)*

(b) (i) Use the trapezium rule with five ordinates (four strips) to find an approximate value for $\int_0^2 2^x \, dx$, giving your answer to three significant figures. *(4 marks)*

(ii) State how you could obtain a better approximation to the value of the integral using the trapezium rule. *(1 mark)*

(c) Describe a geometrical transformation that maps the graph of $y = 2^x$ onto the graph of $y = 2^{x+7} + 3$. *(3 marks)*

(d) The curve $y = 2^{x+k} + 3$ intersects the y-axis at the point $A(0, 8)$.

Show that $k = \log_m n$, where m and n are integers. *(2 marks)*

7 (a) The first four terms of the binomial expansion of $(1 + 2x)^7$ in ascending powers of x are $1 + ax + bx^2 + cx^3$. Find the values of the integers a, b and c. *(4 marks)*

(b) Hence find the coefficient of x^3 in the expansion of $\left(1 - \frac{1}{2}x\right)^2 (1 + 2x)^7$. *(4 marks)*

8 (a) Solve the equation $\tan(x + 52°) = \tan 22°$, giving the values of x in the interval $0° \leqslant x \leqslant 360°$. *(3 marks)*

(b) (i) Show that the equation

$$3 \tan \theta = \frac{8}{\sin \theta}$$

can be written as

$$3 \cos^2 \theta + 8 \cos \theta - 3 = 0$$ *(3 marks)*

(ii) Find the value of $\cos \theta$ that satisfies the equation

$$3 \cos^2 \theta + 8 \cos \theta - 3 = 0$$ *(2 marks)*

(iii) Hence solve the equation

$$3 \tan 2x = \frac{8}{\sin 2x}$$

giving all values of x to the nearest degree in the interval $0° \leqslant x \leqslant 180°$. *(4 marks)*

END OF QUESTIONS

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Surname										
Other Names										
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General Certificate of Education
Advanced Subsidiary Examination
June 2010

Mathematics MPC2

Unit Pure Core 2

Monday 24 May 2010 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

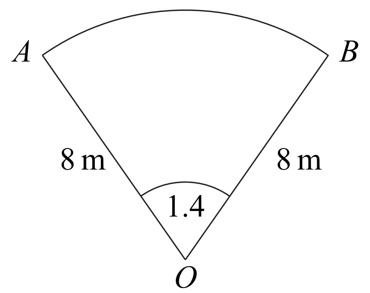
Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions in the spaces provided.

1 The diagram shows a sector OAB of a circle with centre O.



The radius of the circle is 8 m and the angle AOB is 1.4 radians.

(a) Find the area of the sector OAB. *(2 marks)*

(b) (i) Find the perimeter of the sector OAB. *(3 marks)*

(ii) The perimeter of the sector OAB is equal to the circumference of a circle of radius x m. Calculate the value of x to three significant figures. *(2 marks)*

QUESTION PART REFERENCE

Do not write outside the box

2 The nth term of a sequence is u_n.

The sequence is defined by

$$u_{n+1} = 6 + \frac{2}{5}u_n$$

The first term of the sequence is given by $u_1 = 2$.

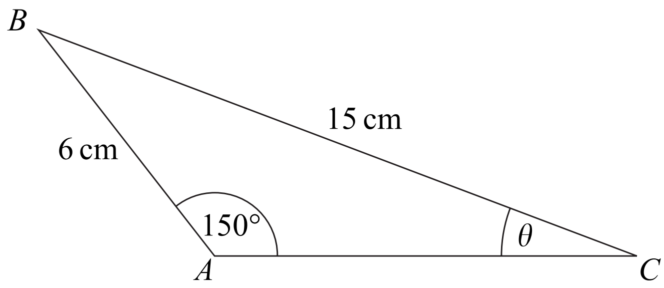
(a) Find the value of u_2 and the value of u_3. *(2 marks)*

(b) The limit of u_n as n tends to infinity is L.

Write down an equation for L and hence find the value of L. *(3 marks)*

QUESTION PART REFERENCE

3 The triangle ABC, shown in the diagram, is such that $AB = 6$ cm, $BC = 15$ cm, angle $BAC = 150°$ and angle $ACB = \theta$.



(a) Show that $\theta = 11.5°$, correct to the nearest 0.1°. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer in cm^2 to three significant figures. *(3 marks)*

QUESTION PART REFERENCE

4 (a) The expression $\left(1 - \dfrac{1}{x^2}\right)^3$ can be written in the form

$$1 + \frac{p}{x^2} + \frac{q}{x^4} - \frac{1}{x^6}$$

Find the values of the integers p and q. *(2 marks)*

(b) (i) Hence find $\displaystyle\int \left(1 - \frac{1}{x^2}\right)^3 \mathrm{d}x$. *(4 marks)*

(ii) Hence find the value of $\displaystyle\int_{\frac{1}{2}}^{1} \left(1 - \frac{1}{x^2}\right)^3 \mathrm{d}x$. *(2 marks)*

QUESTION PART REFERENCE

5 (a) An infinite geometric series has common ratio r.

The first term of the series is 10 and its sum to infinity is 50.

(i) Show that $r = \frac{4}{5}$. *(2 marks)*

(ii) Find the second term of the series. *(2 marks)*

(b) The first and second terms of the geometric series in part **(a)** have the same values as the 4th and 8th terms respectively of an arithmetic series.

(i) Find the common difference of the arithmetic series. *(3 marks)*

(ii) The nth term of the arithmetic series is u_n. Find the value of $\sum_{n=1}^{40} u_n$. *(4 marks)*

QUESTION PART REFERENCE

Do not write outside the box

6 A curve C has the equation

$$y = \frac{x^3 + \sqrt{x}}{x}, \quad x > 0$$

(a) Express $\frac{x^3 + \sqrt{x}}{x}$ in the form $x^p + x^q$. *(3 marks)*

(b) (i) Hence find $\frac{dy}{dx}$. *(2 marks)*

(ii) Find an equation of the normal to the curve C at the point on the curve where $x = 1$. *(4 marks)*

(c) (i) Find $\frac{d^2y}{dx^2}$. *(2 marks)*

(ii) Hence deduce that the curve C has no maximum points. *(2 marks)*

QUESTION PART REFERENCE

Do not write outside the box

7 (a) Sketch the graph of $y = \cos x$ in the interval $0 \leqslant x \leqslant 2\pi$. State the values of the intercepts with the coordinate axes. *(2 marks)*

(b) (i) Given that

$$\sin^2 \theta = \cos \theta (2 - \cos \theta)$$

prove that $\cos \theta = \frac{1}{2}$. *(2 marks)*

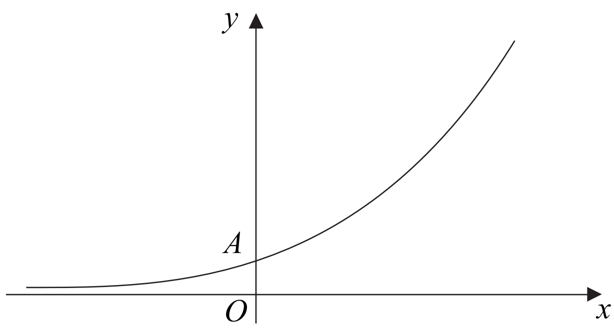
(ii) Hence solve the equation

$$\sin^2 2x = \cos 2x (2 - \cos 2x)$$

in the interval $0 \leqslant x \leqslant \pi$, giving your answers in radians to three significant figures. *(4 marks)*

QUESTION PART REFERENCE

8 The diagram shows a sketch of the curve $y = 2^{4x}$.



The curve intersects the y-axis at the point A.

(a) Find the value of the y-coordinate of A. *(1 mark)*

(b) Use the trapezium rule with six ordinates (five strips) to find an approximate value for $\int_0^1 2^{4x}\,dx$, giving your answer to two decimal places. *(4 marks)*

(c) Describe the geometrical transformation that maps the graph of $y = 2^{4x}$ onto the graph of $y = 2^{4x-3}$. *(2 marks)*

(d) The curve $y = 2^{4x}$ is translated by the vector $\begin{bmatrix} 1 \\ -\frac{1}{2} \end{bmatrix}$ to give the curve $y = g(x)$.

The curve $y = g(x)$ crosses the x-axis at the point Q. Find the x-coordinate of Q. *(4 marks)*

(e) (i) Given that

$$\log_a k = 3\log_a 2 + \log_a 5 - \log_a 4$$

show that $k = 10$. *(3 marks)*

(ii) The line $y = \frac{5}{4}$ crosses the curve $y = 2^{4x-3}$ at the point P. Show that the x-coordinate of P is $\dfrac{1}{4\log_{10} 2}$. *(3 marks)*

QUESTION PART REFERENCE

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General Certificate of Education
Advanced Subsidiary Examination
January 2011

Mathematics MPC2

Unit Pure Core 2

Monday 10 January 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

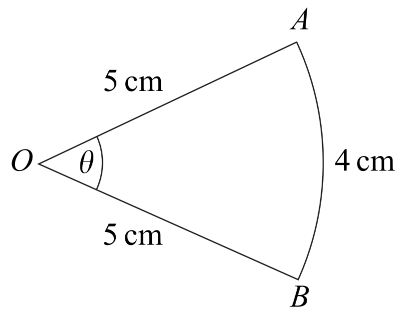
Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

Answer **all** questions in the spaces provided.

1 The diagram shows a sector OAB of a circle with centre O and radius 5 cm.



The angle between the radii OA and OB is θ radians.

The length of the arc AB is 4 cm.

(a) Find the value of θ. *(2 marks)*

(b) Find the area of the sector OAB. *(2 marks)*

QUESTION PART REFERENCE

2 (a) Write down the values of p, q and r given that:

(i) $8 = 2^p$; *(1 mark)*

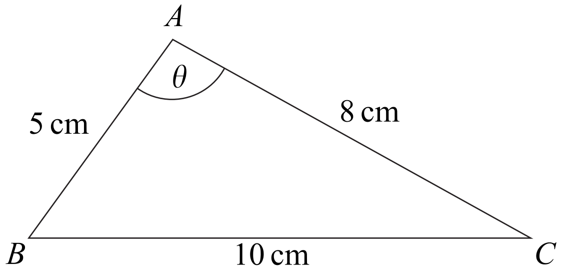
(ii) $\frac{1}{8} = 2^q$; *(1 mark)*

(iii) $\sqrt{2} = 2^r$. *(1 mark)*

(b) Find the value of x for which $\sqrt{2} \times 2^x = \frac{1}{8}$. *(2 marks)*

QUESTION PART REFERENCE

3 The triangle ABC, shown in the diagram, is such that $AB = 5\text{ cm}$, $AC = 8\text{ cm}$, $BC = 10\text{ cm}$ and angle $BAC = \theta$.



(a) Show that $\theta = 97.9°$, correct to the nearest $0.1°$. *(3 marks)*

(b) (i) Calculate the area of triangle ABC, giving your answer, in cm^2, to three significant figures. *(2 marks)*

(ii) The line through A, perpendicular to BC, meets BC at the point D. Calculate the length of AD, giving your answer, in cm, to three significant figures. *(3 marks)*

QUESTION PART REFERENCE

Do not write outside the box

4 (a) Use the trapezium rule with four ordinates (three strips) to find an approximate value for $\int_0^{1.5} \sqrt{27x^3 + 4}\, dx$, giving your answer to three significant figures. *(4 marks)*

(b) The curve with equation $y = \sqrt{27x^3 + 4}$ is stretched parallel to the x-axis with scale factor 3 to give the curve with equation $y = g(x)$. Write down an expression for $g(x)$. *(2 marks)*

QUESTION PART REFERENCE

Do not write outside the box

5 (a) Using the binomial expansion, or otherwise, express $(1-x)^3$ in ascending powers of x. *(2 marks)*

(b) Show that the expansion of

$$(1+y)^4 - (1-y)^3$$

is

$$7y + py^2 + qy^3 + y^4$$

where p and q are constants to be found. *(4 marks)*

(c) Hence find $\int \left[\left(1+\sqrt{x}\right)^4 - \left(1-\sqrt{x}\right)^3 \right] \mathrm{d}x$, expressing each coefficient in its simplest form. *(4 marks)*

QUESTION PART REFERENCE

6 A geometric series has third term 36 and sixth term 972.

(a) (i) Show that the common ratio of the series is 3. *(2 marks)*

(ii) Find the first term of the series. *(2 marks)*

(b) The nth term of the series is u_n.

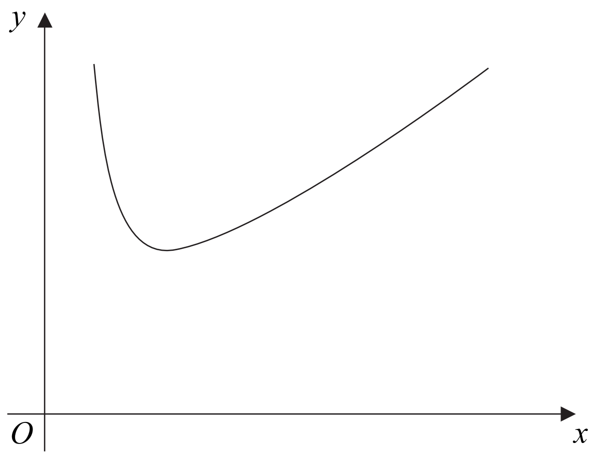
(i) Show that $\sum_{n=1}^{20} u_n = 2(3^{20} - 1)$. *(2 marks)*

(ii) Find the least value of n such that $u_n > 4 \times 10^{15}$. *(3 marks)*

QUESTION PART REFERENCE

Do not write outside the box

7 A curve C is defined for $x > 0$ by the equation $y = x + 3 + \frac{8}{x^4}$ and is sketched below.



(a) Given that $y = x + 3 + \frac{8}{x^4}$, find $\frac{dy}{dx}$. *(3 marks)*

(b) Find an equation of the tangent at the point on the curve C where $x = 1$. *(3 marks)*

(c) The curve C has a minimum point M. Find the coordinates of M. *(4 marks)*

(d) (i) Find $\int \left(x + 3 + \frac{8}{x^4} \right) dx$. *(3 marks)*

(ii) Hence find the area of the region bounded by the curve C, the x-axis and the lines $x = 1$ and $x = 2$. *(2 marks)*

(e) The curve C is translated by $\begin{bmatrix} 0 \\ k \end{bmatrix}$ to give the curve $y = f(x)$. Given that the x-axis is a tangent to the curve $y = f(x)$, state the value of the constant k. *(1 mark)*

QUESTION PART REFERENCE

Do not write outside the box

8 (a) Given that $2\log_k x - \log_k 5 = 1$, express k in terms of x. Give your answer in a form not involving logarithms. *(4 marks)*

(b) Given that $\log_a y = \frac{3}{2}$ and that $\log_4 a = b + 2$, show that $y = 2^p$, where p is an expression in terms of b. *(3 marks)*

QUESTION PART REFERENCE

9 (a) Solve the equation $\tan x = -3$ in the interval $0° \leqslant x \leqslant 360°$, giving your answers to the nearest degree. *(3 marks)*

(b) (i) Given that

$$7\sin^2\theta + \sin\theta\cos\theta = 6$$

show that

$$\tan^2\theta + \tan\theta - 6 = 0$$ *(3 marks)*

(ii) Hence solve the equation $7\sin^2\theta + \sin\theta\cos\theta = 6$ in the interval $0° \leqslant \theta \leqslant 360°$, giving your answers to the nearest degree. *(4 marks)*

QUESTION PART REFERENCE

General Certificate of Education
Advanced Subsidiary Examination
June 2011

Mathematics MPC2

Unit Pure Core 2

Wednesday 18 May 2011 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

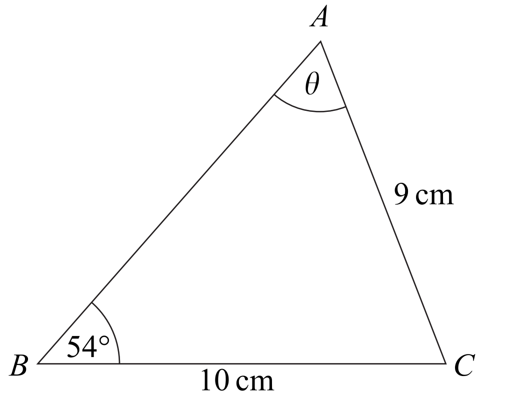
Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

1 The triangle ABC, shown in the diagram, is such that $AC = 9\,\text{cm}$, $BC = 10\,\text{cm}$, angle $ABC = 54°$ and the acute angle $BAC = \theta$.



(a) Show that $\theta = 64°$, correct to the nearest degree. *(3 marks)*

(b) Calculate the area of triangle ABC, giving your answer to the nearest square centimetre. *(3 marks)*

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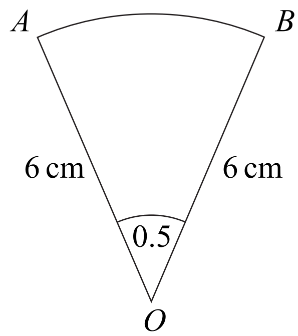
2 The diagram shows a sector OAB of a circle with centre O.



The radius of the circle is 6 cm and the angle $AOB = 0.5$ radians.

(a) Find the area of the sector OAB. *(2 marks)*

(b) (i) Find the length of the arc AB. *(2 marks)*

(ii) Hence show that

$$\text{the perimeter of the sector } OAB = k \times \text{the length of the arc } AB$$

where k is an integer. *(2 marks)*

3 (a) The expression $(2+x^2)^3$ can be written in the form

$$8 + px^2 + qx^4 + x^6$$

Show that $p = 12$ and find the value of the integer q. *(3 marks)*

(b) (i) Hence find $\displaystyle\int \frac{(2+x^2)^3}{x^4}\,\mathrm{d}x$. *(5 marks)*

(ii) Hence find the exact value of $\displaystyle\int_1^2 \frac{(2+x^2)^3}{x^4}\,\mathrm{d}x$. *(2 marks)*

---

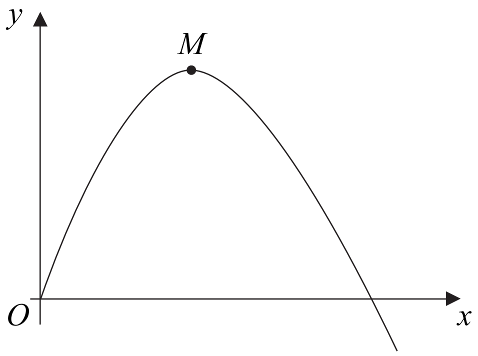
4 (a) Sketch the curve with equation $y = 4^x$, indicating the coordinates of any point where the curve intersects the coordinate axes. *(2 marks)*

(b) Describe the geometrical transformation that maps the graph of $y = 4^x$ onto the graph of $y = 4^x - 5$. *(2 marks)*

(c) (i) Use the substitution $Y = 2^x$ to show that the equation $4^x - 2^{x+2} - 5 = 0$ can be written as $Y^2 - 4Y - 5 = 0$. *(2 marks)*

(ii) Hence show that the equation $4^x - 2^{x+2} - 5 = 0$ has only one real solution. Use logarithms to find this solution, giving your answer to three decimal places. *(4 marks)*

5 The diagram shows part of a curve with a maximum point M.



The curve is defined for $x \geqslant 0$ by the equation

$$y = 6x - 2x^{\frac{3}{2}}$$

(a) Find $\dfrac{dy}{dx}$. *(3 marks)*

(b) (i) Hence find the coordinates of the maximum point M. *(3 marks)*

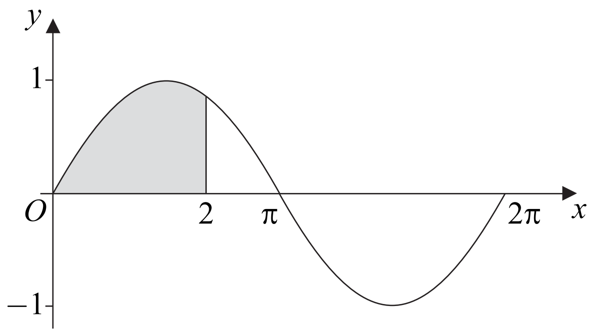
(ii) Write down the equation of the normal to the curve at M. *(1 mark)*

(c) The point $P\left(\frac{9}{4}, \frac{27}{4}\right)$ lies on the curve.

(i) Find an equation of the normal to the curve at the point P, giving your answer in the form $ax + by = c$, where a, b and c are positive integers. *(4 marks)*

(ii) The normals to the curve at the points M and P intersect at the point R. Find the coordinates of R. *(2 marks)*

6 A curve C, defined for $0 \leqslant x \leqslant 2\pi$ by the equation $y = \sin x$, where x is in radians, is sketched below. The region bounded by the curve C, the x-axis from 0 to 2 and the line $x = 2$ is shaded.



(a) The area of the shaded region is given by $\int_0^2 \sin x \, dx$, where x is in radians.

Use the trapezium rule with five ordinates (four strips) to find an approximate value for the area of the shaded region, giving your answer to three significant figures. *(4 marks)*

(b) Describe the geometrical transformation that maps the graph of $y = \sin x$ onto the graph of $y = 2\sin x$. *(2 marks)*

(c) Use a trigonometrical identity to solve the equation

$$2\sin x = \cos x$$

in the interval $0 \leqslant x \leqslant 2\pi$, giving your solutions in radians to three significant figures. *(4 marks)*

---

7 The nth term of a sequence is u_n. The sequence is defined by

$$u_{n+1} = pu_n + q$$

where p and q are constants.

The first two terms of the sequence are given by $u_1 = 60$ and $u_2 = 48$.

The limit of u_n as n tends to infinity is 12.

(a) Show that $p = \frac{3}{4}$ and find the value of q. *(5 marks)*

(b) Find the value of u_3. *(1 mark)*

Turn over ▶

8 Prove that, for all values of x, the value of the expression

$$(3\sin x + \cos x)^2 + (\sin x - 3\cos x)^2$$

is an integer and state its value. *(4 marks)*

---

9 The first term of a geometric series is 12 and the common ratio of the series is $\frac{3}{8}$.

(a) Find the sum to infinity of the series. *(2 marks)*

(b) Show that the sixth term of the series can be written in the form $\frac{3^6}{2^{13}}$. *(3 marks)*

(c) The nth term of the series is u_n.

(i) Write down an expression for u_n in terms of n. *(1 mark)*

(ii) Hence show that

$$\log_a u_n = n\log_a 3 - (3n - 5)\log_a 2$$ *(4 marks)*

END OF QUESTIONS

General Certificate of Education
Advanced Subsidiary Examination
January 2012

Mathematics MPC2

Unit Pure Core 2

Friday 13 January 2012 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

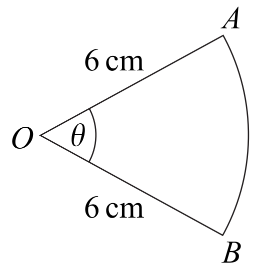
Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer the questions in the spaces provided. Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 The diagram shows a sector OAB of a circle with centre O and radius 6 cm.



The angle between the radii OA and OB is θ radians.

The area of the sector OAB is $21.6\,\text{cm}^2$.

(a) Find the value of θ. *(2 marks)*

(b) Find the length of the arc AB. *(2 marks)*

---

2 (a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for

$$\int_0^4 \frac{2^x}{x+1}\,dx$$

giving your answer to three significant figures. *(4 marks)*

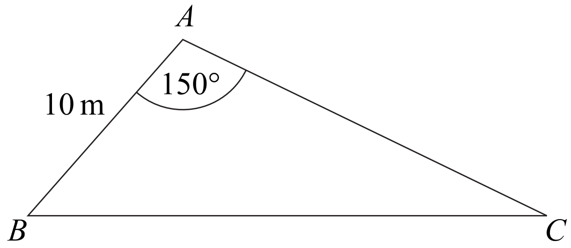
(b) State how you could obtain a better approximation to the value of the integral using the trapezium rule. *(1 mark)*

---

3 (a) Write $\sqrt[4]{x^3}$ in the form x^k. *(1 mark)*

(b) Write $\dfrac{1-x^2}{\sqrt[4]{x^3}}$ in the form $x^p - x^q$. *(2 marks)*

4 The triangle ABC, shown in the diagram, is such that AB is 10 metres and angle BAC is 150°.



The area of triangle ABC is $40\,\text{m}^2$.

(a) Show that the length of AC is 16 metres. *(2 marks)*

(b) Calculate the length of BC, giving your answer, in metres, to two decimal places. *(3 marks)*

(c) Calculate the smallest angle of triangle ABC, giving your answer to the nearest 0.1°. *(3 marks)*

---

5 (a) (i) Describe the geometrical transformation that maps the graph of $y = \left(1 + \frac{x}{3}\right)^6$ onto the graph of $y = (1 + 2x)^6$. *(2 marks)*

(ii) The curve $y = \left(1 + \frac{x}{3}\right)^6$ is translated by the vector $\begin{bmatrix} 3 \\ 0 \end{bmatrix}$ to give the curve $y = \text{g}(x)$. Find an expression for $\text{g}(x)$, simplifying your answer. *(2 marks)*

(b) The first four terms in the binomial expansion of $\left(1 + \frac{x}{3}\right)^6$ are $1 + ax + bx^2 + cx^3$.

Find the values of the constants a, b and c, giving your answers in their simplest form. *(4 marks)*

Turn over ▶

6 An arithmetic series has first term a and common difference d.

The sum of the first 25 terms of the series is 3500.

(a) Show that $a + 12d = 140$. *(3 marks)*

(b) The fifth term of this series is 100.

Find the value of d and the value of a. *(4 marks)*

(c) The nth term of this series is u_n. Given that

$$33\left(\sum_{n=1}^{25} u_n - \sum_{n=1}^{k} u_n\right) = 67\sum_{n=1}^{k} u_n$$

find the value of $\sum_{n=1}^{k} u_n$. *(3 marks)*

---

7 (a) Sketch the graph of $y = \frac{1}{2^x}$, indicating the value of the intercept on the y-axis. *(2 marks)*

(b) Use logarithms to solve the equation $\frac{1}{2^x} = \frac{5}{4}$, giving your answer to three significant figures. *(3 marks)*

(c) Given that

$$\log_a(b^2) + 3\log_a y = 3 + 2\log_a\left(\frac{y}{a}\right)$$

express y in terms of a and b.

Give your answer in a form not involving logarithms. *(5 marks)*

---

8 (a) Given that $2\sin\theta = 7\cos\theta$, find the value of $\tan\theta$. *(2 marks)*

(b) (i) Use an appropriate identity to show that the equation

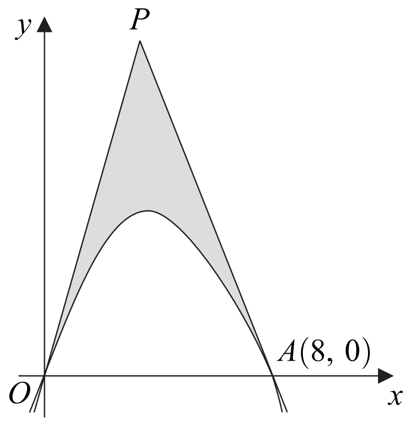
$$6\sin^2 x = 4 + \cos x$$

can be written as

$$6\cos^2 x + \cos x - 2 = 0$$ *(2 marks)*

(ii) Hence solve the equation $6\sin^2 x = 4 + \cos x$ in the interval $0° < x < 360°$, giving your answers to the nearest degree. *(6 marks)*

9 The diagram shows part of a curve crossing the x-axis at the origin O and at the point $A(8, 0)$. Tangents to the curve at O and A meet at the point P, as shown in the diagram.



The curve has equation

$$y = 12x - 3x^{\frac{5}{3}}$$

(a) Find $\dfrac{dy}{dx}$. *(2 marks)*

(b) (i) Find the value of $\dfrac{dy}{dx}$ at the point O and hence write down an equation of the tangent at O. *(2 marks)*

(ii) Show that the equation of the tangent at $A(8, 0)$ is $y + 8x = 64$. *(3 marks)*

(c) Find $\displaystyle\int \left(12x - 3x^{\frac{5}{3}}\right) dx$. *(3 marks)*

(d) Calculate the area of the shaded region bounded by the curve from O to A and the tangents OP and AP. *(7 marks)*

General Certificate of Education
Advanced Subsidiary Examination
June 2012

Mathematics MPC2

Unit Pure Core 2

Wednesday 16 May 2012 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 The arithmetic series

$$23 + 32 + 41 + 50 + \ldots + 2534$$

has 280 terms.

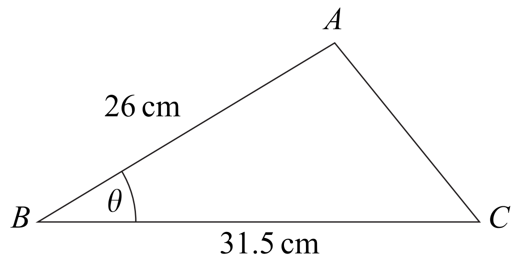
(a) Write down the common difference of the series. *(1 mark)*

(b) Find the 100th term of the series. *(2 marks)*

(c) Find the sum of the 280 terms of the series. *(2 marks)*

---

2 The triangle ABC, shown in the diagram, is such that $AB = 26\text{ cm}$ and $BC = 31.5\text{ cm}$.



The acute angle ABC is θ, where $\sin\theta = \frac{5}{13}$.

(a) Calculate the area of triangle ABC. *(2 marks)*

(b) Find the exact value of $\cos\theta$. *(1 mark)*

(c) Calculate the length of AC. *(3 marks)*

---

3 (a) Expand $\left(x^{\frac{3}{2}} - 1\right)^2$. *(2 marks)*

(b) Hence find $\int \left(x^{\frac{3}{2}} - 1\right)^2 \mathrm{d}x$. *(3 marks)*

(c) Hence find the value of $\int_1^4 \left(x^{\frac{3}{2}} - 1\right)^2 \mathrm{d}x$. *(2 marks)*

4 The nth term of a geometric series is u_n, where $u_n = 48\left(\frac{1}{4}\right)^n$.

(a) Find the value of u_1 and the value of u_2. *(2 marks)*

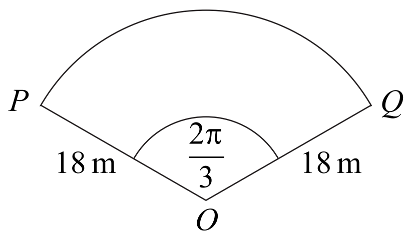
(b) Find the value of the common ratio of the series. *(1 mark)*

(c) Find the sum to infinity of the series. *(2 marks)*

(d) Find the value of $\sum_{n=4}^{\infty} u_n$. *(3 marks)*

---

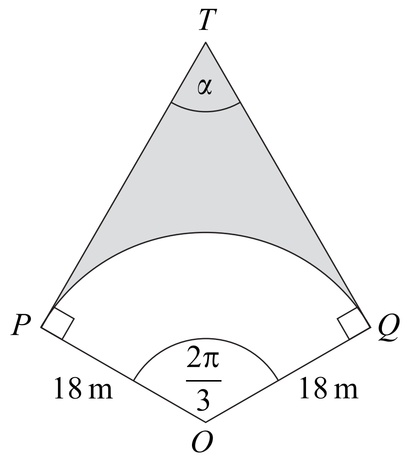
5 The diagram shows a sector OPQ of a circle with centre O.



The radius of the circle is 18 m and the angle POQ is $\frac{2\pi}{3}$ radians.

(a) Find the length of the arc PQ, giving your answer as a multiple of π. *(2 marks)*

(b) The tangents to the circle at the points P and Q meet at the point T, and the angles TPO and TQO are both right angles, as shown in the diagram below.



(i) Angle $PTQ = \alpha$ radians. Find α in terms of π. *(1 mark)*

(ii) Find the area of the shaded region bounded by the arc PQ and the tangents TP and TQ, giving your answer to three significant figures. *(6 marks)*

Turn over ▶

6 At the point (x, y), where $x > 0$, the gradient of a curve is given by

$$\frac{dy}{dx} = 3x^2 - \frac{4}{x^2} - 11$$

The point $P(2, 1)$ lies on the curve.

(a) (i) Verify that $\frac{dy}{dx} = 0$ when $x = 2$. *(1 mark)*

(ii) Find the value of $\frac{d^2y}{dx^2}$ when $x = 2$. *(4 marks)*

(iii) Hence state whether P is a maximum point or a minimum point, giving a reason for your answer. *(1 mark)*

(b) Find the equation of the curve. *(4 marks)*

---

7 It is given that $(\tan\theta + 1)(\sin^2\theta - 3\cos^2\theta) = 0$.

(a) Find the possible values of $\tan\theta$. *(4 marks)*

(b) Hence solve the equation $(\tan\theta + 1)(\sin^2\theta - 3\cos^2\theta) = 0$, giving all solutions for θ, in degrees, in the interval $0° \leqslant \theta \leqslant 180°$. *(3 marks)*

---

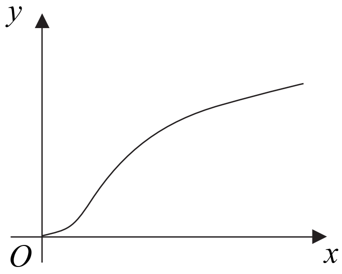
8 (a) Sketch the curve with equation $y = 7^x$, indicating the coordinates of any point where the curve intersects the coordinate axes. *(2 marks)*

(b) The curve C_1 has equation $y = 7^x$.
The curve C_2 has equation $y = 7^{2x} - 12$.

(i) By forming and solving a quadratic equation, prove that the curves C_1 and C_2 intersect at exactly one point. State the y-coordinate of this point. *(4 marks)*

(ii) Use logarithms to find the x-coordinate of the point of intersection of C_1 and C_2, giving your answer to three significant figures. *(2 marks)*

9 The diagram shows part of a curve whose equation is $y = \log_{10}(x^2 + 1)$.



(a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for

$$\int_0^1 \log_{10}(x^2 + 1)\,dx$$

giving your answer to three significant figures. *(4 marks)*

(b) The graph of $y = 2\log_{10} x$ can be transformed into the graph of $y = 1 + 2\log_{10} x$ by means of a translation. Write down the vector of the translation. *(1 mark)*

(c) (i) Show that $\log_{10}(10x^2) = 1 + 2\log_{10} x$. *(2 marks)*

(ii) Show that the graph of $y = 2\log_{10} x$ can also be transformed into the graph of $y = 1 + 2\log_{10} x$ by means of a **stretch**, and describe the stretch. *(4 marks)*

(iii) The curve with equation $y = 1 + 2\log_{10} x$ intersects the curve $y = \log_{10}(x^2 + 1)$ at the point P. Given that the x-coordinate of P is positive, find the gradient of the line OP, where O is the origin. Give your answer in the form $\log_{10}\left(\frac{a}{b}\right)$, where a and b are integers. *(4 marks)*

General Certificate of Education
Advanced Subsidiary Examination
January 2013

Mathematics MPC2

Unit Pure Core 2

Monday 14 January 2013 9.00 am to 10.30 am

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

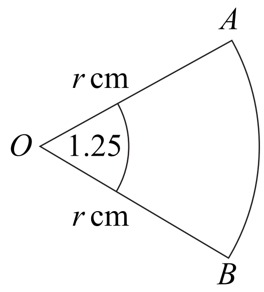
Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 The diagram shows a sector OAB of a circle with centre O and radius r cm.



The angle AOB is 1.25 radians. The perimeter of the sector is 39 cm.

(a) Show that $r = 12$. *(3 marks)*

(b) Calculate the area of the sector OAB. *(2 marks)*

---

2 (a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for

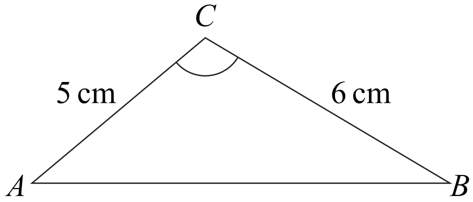
$$\int_1^5 \frac{1}{x^2+1}\,dx$$

giving your answer to three significant figures. *(4 marks)*

(b) (i) Find $\int \left(x^{-\frac{3}{2}} + 6x^{\frac{1}{2}} \right) dx$, giving the coefficient of each term in its simplest form. *(3 marks)*

(ii) Hence find the value of $\int_1^4 \left(x^{-\frac{3}{2}} + 6x^{\frac{1}{2}} \right) dx$. *(2 marks)*

3 The diagram shows a triangle ABC.



The lengths of AC and BC are 5 cm and 6 cm respectively.

The area of triangle ABC is 12.5 cm^2, and angle ACB is **obtuse**.

(a) Find the size of angle ACB, giving your answer to the nearest 0.1°. *(3 marks)*

(b) Find the length of AB, giving your answer to two significant figures. *(3 marks)*

---

4 Given that

$$\log_a N - \log_a x = \frac{3}{2}$$

express x in terms of a and N, giving your answer in a form not involving logarithms. *(3 marks)*

---

5 The point $P(2, 8)$ lies on a curve, and the point M is the only stationary point of the curve.

The curve has equation $y = 6 + 2x - \dfrac{8}{x^2}$.

(a) Find $\dfrac{dy}{dx}$. *(3 marks)*

(b) Show that the normal to the curve at the point $P(2, 8)$ has equation $x + 4y = 34$. *(3 marks)*

(c) (i) Show that the stationary point M lies on the x-axis. *(3 marks)*

(ii) Hence **write down** the equation of the tangent to the curve at M. *(1 mark)*

(d) The tangent to the curve at M and the normal to the curve at P intersect at the point T. Find the coordinates of T. *(2 marks)*

Turn over ▶

6 (a) A geometric series begins $420 + 294 + 205.8 + ...$

(i) Show that the common ratio of the series is 0.7. *(1 mark)*

(ii) Find the sum to infinity of the series. *(2 marks)*

(iii) Write the nth term of the series in the form $p \times q^n$, where p and q are constants. *(2 marks)*

(b) The first term of an arithmetic series is 240 and the common difference of the series is -8.

The nth term of the series is u_n.

(i) Write down an expression for u_n. *(1 mark)*

(ii) Given that $u_k = 0$, find the value of $\sum_{n=1}^{k} u_n$. *(4 marks)*

---

7 (a) Describe a geometrical transformation that maps the graph of $y = 4^x$ onto the graph of $y = 3 \times 4^x$. *(2 marks)*

(b) Sketch the curve with equation $y = 3 \times 4^x$, indicating the value of the intercept on the y-axis. *(2 marks)*

(c) The curve with equation $y = 4^{-x}$ intersects the curve $y = 3 \times 4^x$ at the point P. Use logarithms to find the x-coordinate of P, giving your answer to three significant figures. *(5 marks)*

---

8 (a) Expand $\left(1 + \frac{4}{x}\right)^2$. *(1 mark)*

(b) The first four terms of the binomial expansion of $\left(1 + \frac{x}{4}\right)^8$ in ascending powers of x are $1 + ax + bx^2 + cx^3$. Find the values of the constants a, b and c. *(4 marks)*

(c) Hence find the coefficient of x in the expansion of $\left(1 + \frac{4}{x}\right)^2 \left(1 + \frac{x}{4}\right)^8$. *(4 marks)*

9 (a) Write down the two solutions of the equation $\tan(x + 30°) = \tan 79°$ in the interval $0° \leqslant x \leqslant 360°$. *(2 marks)*

(b) Describe a single geometrical transformation that maps the graph of $y = \tan x$ onto the graph of $y = \tan(x + 30°)$. *(2 marks)*

(c) (i) Given that $5 + \sin^2\theta = (5 + 3\cos\theta)\cos\theta$, show that $\cos\theta = \frac{3}{4}$. *(5 marks)*

(ii) **Hence** solve the equation $5 + \sin^2 2x = (5 + 3\cos 2x)\cos 2x$ in the interval $0 < x < 2\pi$, giving your values of x in radians to three significant figures. *(3 marks)*

General Certificate of Education
Advanced Subsidiary Examination
June 2013

Mathematics MPC2

Unit Pure Core 2

Monday 13 May 2013 1.30 pm to 3.00 pm

For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Time allowed
- 1 hour 30 minutes

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
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- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

1 A geometric series has first term 80 and common ratio $\frac{1}{2}$.

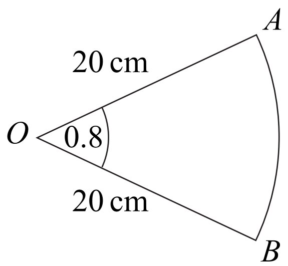
(a) Find the third term of the series. *(1 mark)*

(b) Find the sum to infinity of the series. *(2 marks)*

(c) Find the sum of the first 12 terms of the series, giving your answer to two decimal places. *(2 marks)*

---

2 The diagram shows a sector *OAB* of a circle with centre *O*.

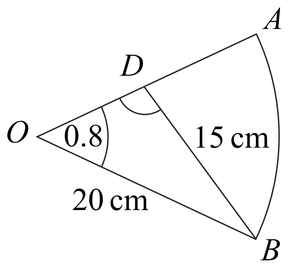


The radius of the circle is 20 cm and the angle $AOB = 0.8$ radians.

(a) Find the length of the arc *AB*. *(2 marks)*

(b) Find the area of the sector *OAB*. *(2 marks)*

(c) A line from *B* meets the radius *OA* at the point *D*, as shown in the diagram below.



The length of *BD* is 15 cm. Find the size of the **obtuse** angle *ODB*, in **radians**, giving your answer to three significant figures. *(4 marks)*

3 (a) (i) Using the binomial expansion, or otherwise, express $(2+y)^3$ in the form $a+by+cy^2+y^3$, where a, b and c are integers. *(2 marks)*

(ii) Hence show that $(2+x^{-2})^3+(2-x^{-2})^3$ can be expressed in the form $p+qx^{-4}$, where p and q are integers. *(3 marks)*

(b) (i) Hence find $\int\left[(2+x^{-2})^3+(2-x^{-2})^3\right]\mathrm{d}x$. *(2 marks)*

(ii) Hence find the value of $\int_1^2\left[(2+x^{-2})^3+(2-x^{-2})^3\right]\mathrm{d}x$. *(2 marks)*

---

4 (a) Sketch the graph of $y=9^x$, indicating the value of the intercept on the y-axis. *(2 marks)*

(b) Use logarithms to solve the equation $9^x=15$, giving your value of x to three significant figures. *(2 marks)*

(c) The curve $y=9^x$ is reflected in the y-axis to give the curve with equation $y=\mathrm{f}(x)$. Write down an expression for $\mathrm{f}(x)$. *(1 mark)*

---

5 (a) Use the trapezium rule with five ordinates (four strips) to find an approximate value for $\int_0^2\sqrt{8x^3+1}\,\mathrm{d}x$, giving your answer to three significant figures. *(4 marks)*

(b) Describe the single transformation that maps the graph of $y=\sqrt{8x^3+1}$ onto the graph of $y=\sqrt{x^3+1}$. *(2 marks)*

(c) The curve with equation $y=\sqrt{x^3+1}$ is translated by $\begin{bmatrix}2\\-0.7\end{bmatrix}$ to give the curve with equation $y=\mathrm{g}(x)$. Find the value of g(4). *(3 marks)*

6 A curve has the equation

$$y = \frac{12 + x^2\sqrt{x}}{x}, \quad x > 0$$

(a) Express $\dfrac{12 + x^2\sqrt{x}}{x}$ in the form $12x^p + x^q$. *(3 marks)*

(b) (i) Hence find $\dfrac{dy}{dx}$. *(2 marks)*

(ii) Find an equation of the normal to the curve at the point on the curve where $x = 4$. *(4 marks)*

(iii) The curve has a stationary point P. Show that the x-coordinate of P can be written in the form 2^k, where k is a rational number. *(3 marks)*

---

7 The nth term of a sequence is u_n. The sequence is defined by

$$u_{n+1} = pu_n + q$$

where p and q are constants.

The first two terms of the sequence are given by $u_1 = 96$ and $u_2 = 72$.

The limit of u_n as n tends to infinity is 24.

(a) Show that $p = \frac{2}{3}$. *(4 marks)*

(b) Find the value of u_3. *(2 marks)*

---

8 (a) Given that $\log_a b = c$, express b in terms of a and c. *(1 mark)*

(b) By forming a quadratic equation, show that there is only one value of x which satisfies the equation $2\log_2(x + 7) - \log_2(x + 5) = 3$. *(6 marks)*

9 (a) (i) On the axes given below, sketch the graph of $y = \tan x$ for $0° \leqslant x \leqslant 360°$. *(3 marks)*

(ii) Solve the equation $\tan x = -1$, giving all values of x in the interval $0° \leqslant x \leqslant 360°$. *(2 marks)*

(b) (i) Given that $6 \tan\theta \sin\theta = 5$, show that $6\cos^2\theta + 5\cos\theta - 6 = 0$. *(3 marks)*

(ii) **Hence** solve the equation $6 \tan 3x \sin 3x = 5$, giving all values of x to the nearest degree in the interval $0° \leqslant x \leqslant 180°$. *(6 marks)*